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# FLEXIBLE DESIGN OF AN OPTICAL WAVEGUIDE THROUGH EVOLUTIONARY COMPUTATION

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**Abstract** - An evolutionary-computation-based method is presented for designing an Antiresonant Reflecting Optical Waveguide, one of the constituting parts of an optical filter for communication networks. The key issue in the design is the problem of finding the best configuration for its component layers, so as to achieve maximum filtering. It is argued that the evolutionary approach is more appealing than another method currently available in the literature, in that it is far more flexible, by supporting an arbitrary number of materials to be chosen from and an arbitrary choice of the layer thicknesses; by allowing both the latter design constraints to be independently handled by the algorithm; by its reliance on separating the optimization process from the mathematical model of the system being optimized; and by the fact that it imposes no restriction in the number of layers that can be used. The evolutionary method is exemplified in the design of a filter with four layers.

## **1. INTRODUCTION**

Optical fiber networks ensure efficiency and speed of information flow; however, they are not yet consolidated in metropolitan and access networks.

The best way to shorten the distance between technology and the benefits of *passive optical networks* [1] is to increase the number of channels in the wavelength band, together with the placement and use of devices with high selectivity in wavelength. In order to allow an increase in the capacity of optical systems, it is necessary to increase the number of channels in the wavelength band of the optical amplifiers, which is only possible with the use of devices such as the optical filters with high selectivity in wavelength.

One advantage of this technology is the success obtained with multiplexing in wavelength, in addition to the fact that these types of network support both *broadcast* and *point-to-point* services [2].

In tune with the latter, an approach to designing the ARROW waveguide – Antiresonant Reflecting Optical Waveguide – of a *dropping*-type optical filter, of a communication network, is presented here. The ARROW waveguide is an integral part of an optical filter that is also composed of a *D*-type fiber optic and a double Bragg reflector (DBR) presented in [3].

Our approach relies on evolutionary computation [5], used as the optimization procedure for designing the best configuration of the filter's component layers, so that it can achieve its maximum filtering performance. These filter layers constitute a set of adjoint materials with low refraction index, stacked over a semiconductor substratum.

The present approach is exemplified in the project of a filter with four layers, in a simplification of the seven-layer waveguide discussed in [3]. In spite of relying on the simplified model, our approach is potentially more flexible than that presented in [3], while being able to abide by the constraints of existing construction technology of the waveguide. Among other points, such flexibility is implemented by allowing the choice of the materials that make up each layer (by explicitly handling their refraction indices), and by allowing arbitrary choice of the thickness of each layer.

In the next section the dropping filter is introduced, with a description of its essential parts. Section 3 then describes the evolutionary algorithm that was developed for the problem, which uses standard techniques for the operations of recombination, proportional selection for reproduction, and elitism, but not for mutation, which is shown to be adaptive [6]. In the subsequent section the results of the evolutionary algorithm, and then the true design experiment.

The article finishes with some concluding remarks on the results, and general comments on the perspectives of using an evolutionary approach in the problem at issue. As the list of references suggest, this paper is indeed the first one in the topic in English; all the others, [3, 6, 11], have been originally written in Portuguese.

## 2. DROPPING FILTER

Figure 1 illustrates a diagrammatic sketch of the full optical filter with the identification of its components. The function of the filter is to receive several wavelengths ( $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$ ,  $\lambda_4$ ) and redirect the wavelength to which it has been configured ( $\lambda_2$  for instance), using the double Bragg reflector, that selects a wavelength according to its resonance condition. The selected wavelength then goes through the ARROW waveguide that, by inducing very

high power losses by *leakage*, entails the wavelength to be captured by a photo-detector located at waveguide substratum [3].



Figure 1. Dropping filter [3].

## 2.1. D-type Fiber

This fiber has a geometry that resembles a lying D letter, what allows its adhesion more easily to the other materials. It has smaller losses by insertion in the optical link when compared to other types of fibers, and does not possess the same propagation constants (due to imperfections caused by lateral pressure, non-circular geometry of its core, or variations in the refraction index profile).

The fundamental propagation mode in the fiber is composed of two degenerate modes, very similar to each other, except by the fact that their polarization plans are orthogonal. The horizontal polarization of the fundamental mode is represented by the  $TE_0$  mode, and the vertical polarization by the  $TM_0$  mode.

The optical fiber used herein has the characteristics presented in Table 1, where  $\eta$  stands for refraction index (of the fiber's core or clad), *r* is its radius of the core, and *t* is its thickness (including the clad); the table also shows the wavelength  $\lambda$  of the carrier signal to be used, which is part of the specification of the system being constructed.

Table 1: The D-type optical fiber.

η (clad)	η (core)	r (core)	t	λ
1.458	1.465	4µm	1µm	1550nm

## 2.2. The Double Bragg Reflector (DBR)

The double Bragg reflector introduces a phase of  $\pi/2$  between 2 roundtrips (passages) of the Bragg network, turning it into a resonant cavity of length L<sub>S</sub> (as depicted in Figure 1), and entailing a transmission peak to appear in the center of the reflection spectrum of the Bragg network.

This transmission peak happens to be very narrow, giving the filter a high extinction rate. The extinction rate is the difference, in the measured damping, between the transmitted and reflected wavelengths, which is very high in the present case; it is such a feature, together with a narrow bandwidth, that accredits the filter for passive optical networks.

#### 2.3. The ARROW Waveguide

Figure 2 shows a longitudinal cut of the layers of the ARROW waveguide, where the *D*-type fiber is attached to a multi-layer structure constituted of low refraction index materials.

In this device, there is a mirror separating the semiconductor substratum and the low refraction index layers above it. In other words, an extremely thin layer of high refraction index prevent the optical energy confined to the waveguide core from escaping to the semiconducting substratum, resulting in considerable loss of guided optical power, since light tends to follow the higher refraction index path. This escaping effect of the optical energy is what known as *leakage*.

## **3. THE EVOLUTIONARY APPROACH**

#### **3.1. Introduction**

Evolutionary algorithms are search methods gleaned from biological evolution [7]. They explore the problem solution space, starting from a random sampling of possible candidates of the solution, then, using operators inspired by biological evolution, they search for the most promising points of the space.

This search method is particularly well suited for problems where the search space is very large, the mathematical formulation of the problem solution leads to non-differentiable or highly non-linear functions, or when no direct formulation of their solution is available [8].



Figure 2. Layers of an ARROW waveguide [3].

Evolutionary computation is based on four fundamental processes: reproduction, random variation, competition and selection, within a population of possible problem solutions [6]. Basically, the following steps are taken: an initial population is generated with individuals representing candidate solutions; each individual then receives a score, corresponding to its ability to solve the problem at issue (that is, its fitness to the environment); reproduction occurs by favouring the most fitted parents; and, genetic operators, usually, recombination and mutation, modify parents and their offspring, also according to their fitness. This process goes on through the generations, yielding progressively more fitted individuals, up to the point that an individual appears that constitute a satisfactory solution of the problem at issue. Naturally, evolutionary computation methods require the representation of a point in the space of candidate solutions, and the definition of an objective function that is meant to guide the search [9].

#### 3.2. Representation of an Individual

For the problem being tackled here the representation of each individual (candidate solution, or chromosome) was defined as a sequence of values, its genes (the elements of this sequence) representing the values of the refraction index or the thickness of each layer, and the initial value of the longitudinal propagation constants of the modes involved in the wave propagation inside the waveguide. As for the latter, both the real and imaginary parts are represented, respectively by  $\beta_r$  and  $\beta_{i \text{ in}}$ , as Figure 3 shows.

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where:

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\begin{split} n_1 &= 1.500 \\ n_2 &\in \{1.458, 1.465, 1.520, 1.550, 1.600\} \\ n_3 &\in \{1.458, 1.465, 1.520, 1.550, 1.600\} \\ n_4 &= 3.500 \\ t_1 &= 0 \\ t_2 &\in [1.000 \times 10^{-6}, 1.700 \times 10^{-6}] \\ t_3 &\in [1.000 \times 10^{-6}, 3.000 \times 10^{-6}] \\ t_4 &= 3.500 \times 10^{-6} \\ k_0 &= (2\pi)/\lambda \text{ and } \lambda = 1550 \text{nm} \\ \text{The real component } \beta_r \text{ is such that:} \\ &\text{ If } n_1 > n_3 \text{ then } \beta_r \in [n_1 \, k_0 \,, n_2 \, k_0] \\ &\text{ If inginary component } \beta_{i\_in} \text{ is such that:} \\ &\beta_{i\_in} \in [0, 0.007 \times k_0] \end{split}
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Figure 3. Chromosome representation.

Notice that while  $\beta_{i_{in}}$  is a parameter of the representation of an individual, another  $\beta_i$  (to be explained below is the actual fitness value that guides evolution.

#### **3.3. Evaluation Function**

The fitness function derives from the analysis of the multi-layer structure of the waveguide, while taking into account the refraction indexes of each layer, their thicknesses, and the longitudinal propagation at the layer inputs, both their real and imaginary parts.

This gives the mathematical model that, from a set of values of its independent variables, yields the corresponding leakage. In other words, this mathematical process acts as the forward model associated with the actual inverse problem that we want to solve, namely, finding out the best parameter value setting that minimizes leakage.



Figure 4. Input/output variables of the evaluation function.

Figure 4 represents the forward model that is used for evaluating a candidate design. The fitness value is the imaginary longitudinal propagation value ( $\beta_i$ ), analytically worked out from the refraction indexes of the input layers, their thicknesses, and the values of  $\beta_r$  and  $\beta_{i_in}$  (respectively, the real and imaginary terms that characterize the traveling wave in the D fiber). This objective function then entails that the better an individual in the population, the closest to zero its associated  $\beta_i$  is (with zero being the global minimum, that is, the ideal solution).

That forward model was obtained out of [3], by simplifying the model therein, from seven to four layers. The simplification was required due to technicalities involved in the original model, and defines a limitation of the current results; we get back to this issue in the last section of the paper.

The remainder of this section provides the details about how the evaluation function is derived; the reader who is only willing to have an overall view of the whole process described herein can skip directly to the next section.

The derivation of the evaluation function is based on the *transfer matrix technique* [3] which is suitable to determine the propagation constant of multilayered dielectric waveguides. So, assuming that the multilayered structures are perpendicular to the y direction (see Figure 1), and the field propagation is in the z direction, the y component of the electric field, for the propagating TE modes, in the j<sup>th</sup> layer, obeys the wave equation

$$\frac{\partial^2}{\partial y^2} E_{x,j}(y) - \left(\beta^2 - k_0^2 n_j^2\right) E_{x,j} = 0$$
<sup>(1)</sup>

where  $\beta$  is the longitudinal propagation constant of the modes,  $k_0$  is the propagation constant in free space and  $n_j$  is the refraction index of the j<sup>th</sup> layer. The solution of eqn. (1) for the j<sup>th</sup> layer is

$$E_{i} = A_{i}e^{a_{j}(y-t_{j})} + B_{i}e^{-a_{j}(y-t_{j})}$$
(2)

where  $\alpha_j = \sqrt{\beta^2 - k_0^2 n_j^2}$ ,  $t_j$  is the position of the j<sup>th</sup> layer, and  $A_j$  and  $B_j$  are constants. In order to respect the field boundary conditions, the transversal electric field components and its derivatives have to be continuous at the interfaces between layers. Hence, for the j<sup>th</sup> layer it follows that:

$$E_{j}(t_{j}) = E_{j+1}(t_{j}) \tag{3}$$

$$\frac{\partial E_{j}(t_{j})}{\partial y} = \frac{\partial E_{j+1}(t_{j})}{\partial y}$$
(4)

By applying the boundary conditions at the interface of the  $j^{th}$  and  $(j+1)^{th}$  layers, the following relation between the field coefficients is obtained:

$$\begin{bmatrix} A_j \\ B_j \end{bmatrix} = T_{j+1} \begin{bmatrix} A_{j+1} \\ B_{j+1} \end{bmatrix}$$
(5)

where

$$T_{j+1} = \begin{bmatrix} \frac{1}{2} \begin{bmatrix} 1 + \begin{pmatrix} \alpha_{j+1} \\ \alpha_j \end{bmatrix} \end{bmatrix} e^{-\delta_{j+1}} & \frac{1}{2} \begin{bmatrix} 1 - \begin{pmatrix} \alpha_{j+1} \\ \alpha_j \end{bmatrix} \end{bmatrix} e^{\delta_{j+1}} \\ \frac{1}{2} \begin{bmatrix} 1 - \begin{pmatrix} \alpha_{j+1} \\ \alpha_j \end{bmatrix} \end{bmatrix} e^{-\delta_{j+1}} & \frac{1}{2} \begin{bmatrix} 1 + \begin{pmatrix} \alpha_{j+1} \\ \alpha_j \end{bmatrix} \end{bmatrix} e^{\delta_{j+1}} \end{bmatrix}$$
(6)

In eqn. (6), the phase  $\delta_{j+1}$  is defined as  $\delta_{j+1} = \alpha_{j+1}d_{j+1}$ , where  $d_{j+1} = t_{j+1} - t_j$  is the thickness of the j+1 layer. Also, the transfer matrices  $T_j$ ,  $T_{j-1}$ ,  $T_{j-2}$ , etc., connecting the field coefficients  $(A_{j-1}, B_{j-1})$  and  $(A_j, B_j)$ ,  $(A_{j-2}, B_{j-2})$  and  $(A_{j-1}, B_{j-1})$ ,  $(A_{j-3}, B_{j-3})$  and  $(A_{j-2}, B_{j-2})$ , etc., respectively, are analogously defined. Furthermore, the total transfer matrix connecting the first layer to the last (that is, the N<sup>th</sup>) layer can be written as

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = T_{wG} \begin{bmatrix} A_N \\ B_N \end{bmatrix}$$
(7)

where  $T_{w_G} = \prod_{k=2}^{N} T_k$ . Additionally, in order to abide by the field radiation condition (that the fields must be evanescent when y approaches  $\pm \infty$ ) the field coefficients must be  $B_1 = 0$  for the first layer and  $A_N = 0$  for the N<sup>th</sup> layer. It is convenient to view the  $T_{w_G}$  matrix as the four-port block matrix

$$T_{WG} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} ,$$
 (7)

where  $t_{11}$  stands for the incident energy in the input port,  $t_{12}$  represents the reflected energy at the input,  $t_{21}$  is the transmitted energy to the output port, and  $t_{22}$  is the incident energy from the output.

In the present case, the propagation constant is a complex number, where the leakage corresponds to the imaginary part ( $\beta_i$ ). The procedure to determine the complex propagation constant is carried out in two steps. First, the real part of the propagation constant ( $\beta_r$ ) is determined, by solving the equation  $t_{11}(\beta) = 0$ . Next, using  $\beta_r$  as an initial guess, the equation  $|t_{12}(\beta)|^2 = 0$  is solved, its root being complex numbers. Finally, these roots are then calculated by the downhill method, which calls for an initial guess for the real and imaginary parts ( $\beta_r$  and  $\beta_{i,in}$ , respectively, as they appear in the representation of a candidate solution, as shown in Figures 3 and 4).

We restrict the analyses to a structure with four layers, and a careful parameter analysis was carried out prior to the integration with the evolutionary algorithm. As result, lower and upper bound values for the layer thicknesses were determined in order to prevent a discontinuous behaviour of  $\beta_i$ .

#### 3.4. Selection and Recombination

An *elitist* selection scheme was adopted, which means that a certain number of individuals of a generation, specified as a percentage of the population size, is directly taken to be part of the subsequent generation.

In addition to the latter that acts as a secondary selection operator the standard proportional schema of *roulette-wheel* was used, as the primary selection operator. Therefore, the probability of an individual being selected grows with its fitness value, guaranteeing a larger reproduction probability to the fittest individuals.

For recombination a *mask*-based scheme was employed for allowing crossover between *homologous* genes. In other words, a random binary mask is generated for each mating pair, in which the positions of the 1-bits define the gene loci whose contents are to be swapped between the parents involved. The maintenance of crossover only between homologous genes is a requirement from the fact that each gene in the representation of the individuals has a precise and specific meaning that has to be preserved across the generations.

#### 3.5. Mutation

After recombination the offspring undergo a mutation process that is based on two separate mechanisms.

The first one simply selects an individual at random, at a predefined rate, the *individual mutation rate*. Once an individual has been chosen to undergo mutation, the second mechanism then plays its role, by defining the mutation probability at the gene level, which is implemented by a *gene mutation rate*. Consequently, even if an individual is selected for mutation through the former mechanism, because of the latter it may not undergo

any change at all. The usage of two mutation parameters materialised in that it allowed for a finer-grained adjustment of the effect of mutation as a whole.

At the gene level mutation can be accomplished in two ways, depending on the gene having a discrete or a real value. If the value is discrete, the gene mutation is implemented by simply drawing any possible value from the discrete domain of the corresponding gene. This scheme is particularly adequate for the gene loci that represent the refraction indexes of the waveguide layers, as they have to be drawn from a handful of materials currently available.

However, if the gene value is a real number – as happens for those that represent the thicknesses of the waveguide layers – the new gene value is taken out of a perturbation of the current value, through a Gaussian distribution. This has the advantage of allowing any new value in the domain to come about, but favouring those, which are closer to the current gene value. Naturally, the limit in the precision of the thickness has to be taken into account, thus constraining the actual perturbation that can be done to the current gene value, but, even then, the resulting value domain is much larger than the domain of refraction indexes.

The Gaussian-based mutation acts by adding or subtracting a random value of a Gaussian distribution that is fitted to the gene locus at issue, centered at the actual gene value (that is, with zero mean), and with the variance taken from the variance of the corresponding gene value across the entire population.

Notice that the Gaussian gene mutation ascribes the process an adaptive nature, a flexibility that is usually absent from genetic algorithms [5, 10], even though present in evolution strategies (see [4] for instance). This adaptiveness entails that, in the initial generations of the evolution these gene mutations occur in larger steps, because the variance of gene values in the population tends to be larger, due to the smaller selective pressure that is typical of this stage of the evolution. But as the evolutionary goes on, selective pressure tends to increase, thus leading to a collapse in the variance of the values at each gene locus [6].

The fact that we used such an adaptive scheme in the present context comes from the fact that it is a viable possibility in a genetic algorithm. Alternative adaptive approaches within evolutionary computation at large could be attempted, such as, say, the covariance matrix adaptation method [4] that is associated with evolution strategies; however, this is beyond present purposes, considering that our main objective herein does not concern finding out the best algorithm for the problem, but, at least at its stage of development, simply providing an evolutionary approach that would overcome the limitations present in the original method [3] used in the problem.

## 4. RESULTS

Firstly, as a baseline for the more general results, experiments were carried out to calibrate the system, so to speak. At this stage existing technology both for the production of the layers and the availability of materials was not taken into account. In other words, limits on the number of decimal points were not imposed for the layer thicknesses, and any value of refraction index was allowed, even if many of them did not have a correspondence to an existing material. In this way, the results at the following order of magnitude were obtained, averaged over 50 runs:

Objective Function	Average Number of Generations
10-1	1
10-2	9
10-3	39
10 <sup>-4</sup>	180
10 <sup>-5</sup>	915

Table 2. Quality of the results as a function of the evolution effort.

Table 2 states that, after about the initial 10 generations, the genetic algorithm has converged to an objective function value at the order of  $10^{-1}$ , and, as generations go on, evolution steadily carries on, yielding progressively better individuals, even though requiring an ever-increasing effort, in an exponential way.

Results at the order of  $1 \times 10^{-5}$  are satisfactory for present purposes, and these have been obtained in 89% of all runs of the algorithm, which is a very robust achievement; in the remaining 11% of the runs, the algorithm failed to reach the required level of quality within the limit of 2000 generations. In contrast, only 6% of the runs achieved a quality level at the order of  $1 \times 10^{-6}$  and only 3% reached  $1 \times 10^{-7}$ , which is a huge decrease in performance of the algorithm. As we will see later, this situation significantly improves when the algorithm is set to run within the technological constraints imposed by the real-world requirements for building the optical filter.

Anyway, for the parameters presented in Table 3, the best result of the objective function was  $75.292 \times 10^{-8}$ , corresponding to the waveguide design represented by Table 4. Notice that the value obtained of the objective function is a very small number, which means that the optical filter leakage has indeed been made down to a minimum.

One should also notice in the parameter values shown in Table 4 that two layers (1 and 4) have been predetermined and, as such, could well be left out of the genome representation for this particular experiment. In fact, layer 4 was made fixed, and layer 1 was suppressed. This was due to a simplification in the forward model we used, that we will be overcome in the follow-up of the research; we get to this matter in the last section.

Population size	100
Crossover rate	100%
Mutation rate per individual	80%
Mutation rate per gene	50%
Maximum number of generations	80000
Elitism rate	10%

Table 3. Parameters of the evolution that yielded the best result.

The baseline experiments carried out allowed concluding about the good degree of robustness of the evolutionary process, in respect to the average quality of the results obtained, the frequency in which they came out, as well as the necessary evolutionary effort for obtaining the results. So, the remaining question was whether the same pattern of occurrences could also be observed if technological constraints would be imposed on the waveguide design, so as to render it realizable.

In order to answer this question, first of all it was defined that a satisfactory result would be a value of the objective function of about 10<sup>-5</sup>; realizability of the project, ensured by the use of existing technology, was then introduced through the reliance upon 3 decimal points for the layer thicknesses (the search precision is implemented in the algorithm as a parameter for each gene, and set at run time), and by allowing search to choose the material of each layer (by means of its refraction index), from the predefined discrete set shown in Figure 3.

$\begin{array}{c cccc} n_1 & 1.500 \\ \hline n_2 & 1.524 \\ \hline n_3 & 1.490 \\ \hline n_4 & 3.500 \\ \hline t_1 & 0 \\ \hline t_2 & 1.153 \times 10^{-6} \\ \hline t_3 & 1.362 \times 10^{-6} \end{array}$		
$\begin{array}{c cccc} n_2 & 1.524 \\ \hline n_3 & 1.490 \\ \hline n_4 & 3.500 \\ \hline t_1 & 0 \\ \hline t_2 & 1.153 \times 10^{-6} \\ \hline t_3 & 1.362 \times 10^{-6} \end{array}$	$n_1$	1.500
$\begin{array}{c ccc} n_3 & 1.490 \\ \hline n_4 & 3.500 \\ \hline t_1 & 0 \\ \hline t_2 & 1.153 \times 10^{-6} \\ \hline t_3 & 1.362 \times 10^{-6} \end{array}$	$n_2$	1.524
$ \begin{array}{c ccc} n_4 & 3.500 \\ \hline t_1 & 0 \\ \hline t_2 & 1.153 \times 10^{-6} \\ \hline t_3 & 1.362 \times 10^{-6} \end{array} $	$n_3$	1.490
$\begin{array}{c ccc} t_1 & 0 \\ \hline t_2 & 1.153 \times 10^{-6} \\ \hline t_3 & 1.362 \times 10^{-6} \end{array}$	$n_4$	3.500
$\begin{array}{c cccc} t_2 & 1.153 \times 10^{-6} \\ \hline t_3 & 1.362 \times 10^{-6} \end{array}$	t <sub>1</sub>	0
$t_3 = 1.362 \times 10^{-6}$	t <sub>2</sub>	1.153×10 <sup>-6</sup>
	t <sub>3</sub>	1.362×10 <sup>-6</sup>
$t_4$ 3.500×10 <sup>-6</sup>	$t_4$	3.500×10 <sup>-6</sup>
β <sub>r</sub> 6.10670×10 <sup>6</sup>	ßr	6.10670×10 <sup>6</sup>
$\beta_{i in}$ 1.81342×10 <sup>4</sup>	ß <sub>i in</sub>	$1.81342 \times 10^4$

Table 4. The best ARROW waveguide design.

With these new, technological constraints imposed, and the evolution parameters shown in Table 5, the best result found for the objective function was  $1.525 \times 10^{-8}$ , corresponding to a waveguide design whose details are presented in Table 6. Notice that the result obtained in this technologically constrained situation is better than the previous, unconstrained version; this can be explained by the fact that now the search space is smaller, making the search more effective, as well as because this experiment was carried out in a more systematic way, that led to a more fine-tuned parameter set.

Upon an analysis of the meaning of the actual values involved in the design achieved, one can conclude that all of them are physically sensible, thus leading to a waveguide design that not only satisfies the criterion of minimizing the leakage of the optical filter, but also allows the actual construction of the waveguide, according to current manufacturing constraints.

Population size	200
Crossover rate	100%
Mutation rate per individual	70%
Mutation rate per gene	60%
Maximum number of generations	3000
Elitism rate	1%

Table 5. Parameters of the evolution that yielded the best result, in the case of a realistic design.

Table 6. The best realistic ARROW	waveguide design.
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$n_1$	1.500
$n_2$	1.550
$n_3$	1.600
$n_4$	3.500
$t_1$	0.000
$t_2$	1.614×10 <sup>-6</sup>
t <sub>3</sub>	1.169×10 <sup>-6</sup>
$t_4$	3.500×10 <sup>-6</sup>
$\beta_{\rm r}$	6.287129×10 <sup>6</sup>
ß <sub>i in</sub>	1.191840×10 <sup>4</sup>

#### **5. FINAL REMARKS**

The method we developed evidences the potential of using evolutionary computation as a tool for designing an optical system. And although there is another method available for the same task [11], the one showed here is more appealing in that it is far more flexible by supporting an arbitrary number of materials to be chosen from and an arbitrary choice of the layer thicknesses, and by allowing both design constraints to be independently handled by the algorithm. Also, our approach abides by the standard separation between the optimization process and the mathematical model of the system, a feature that is not present in [11]. And finally, there is no restriction at all in the present method, in terms of the number of layers involved, as long as the corresponding forward model (used to evaluate the candidate solutions) is available.

The main next objective in the follow-up of this research is to get hold of a seven-layer forward model, so that a comparison of the actual design details can be made between the design that would come out of our approach, with the one described in [3]. In fact, it is fair to bear in mind that the experiments reported herein refer to toy-versions of the problem at issue, a reflection of the preliminary stage of the use of our methodology in a real problem situation. However, our point here is not the comparison of our results in a real-world problem, but rather, to make the case that the method we are pursuing brings naturally with it various advantages over the current competing method in the literature, in terms of the flexibilities we laid out above. It is also fair to bear in mind that we are not making a case for our evolutionary algorithm over other potentially useful optimisation techniques, say, simulated annealing, tabu search, etc.; the matter of which of them would yield the most appropriate results for the problem being tackled is undoubtedly open at the moment.

But as a measure of the computational gain of our evolutionary approach, over the exhaustive design, can be readily made. First consider that it can be shown that a lower bound for the search space size of the problem instance discussed in the paper is of about  $1 \times 10^{12}$  points [6], a figure obtained disregarding the value range of  $\beta_r$ . Now, running the evolutionary algorithm with a population of 100 individuals, through 1000 generations, this guarantees a good quality level at the order of  $10^{-5}$ , and entails a computational effort of about  $1 \times 10^{5}$  evaluations points in the space, which is a significantly smaller effort than enumeration of the entire space.

An issue that has been left out from the presentation so far has to do with the possible impact on to the waveguide design, of the approximations in the mathematical modeling of the filter and the imperfections in its physical realization. This happened so because the forward model we used did not explicitly embed such aspects in its formulation; hence, in [3] this issue has also been left out.

However, one way we can go about it – and, in fact, this is a definite true requirement before an actual waveguide can come into existence – is introducing synthetic noise at the output of the evaluation function. By adding Gaussian noise, with zero mean, and 5 values of standard deviations, from 2% of the mean value upwards, and averaging the results over 20 runs of each noise level, the following pattern came about: with up to

4% noise the evolutionary process remained as robust as in the noiseless case; from more than 4% up to 10%, a progressive decrease in performance became apparent; and with more than 10% noise, the evolutionary process broke down. This conforms with the usual experience with many completely distinct inverse problems that evolutionary algorithms have shown a fair amount of robustness, usually higher than the one obtained through gradient-based optimization procedures, such as the one used in [11]. Nonetheless, a final comparison in our case is yet to be made.

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